

THE BF FORMALISM FOR YANG-MILLS THEORY AND THE 't HOOFT ALGEBRA

MAURIZIO MARTELLINI[†] and MAURO ZENI

Dipartimento di Fisica, Università di Milano and I.N.F.N. - Sezione di Milano.

[†] *Landau Network at "Centro Volta", Como, ITALY*

The deformation of a topological field theory, namely the pure BF theory, gives the first order formulation of Yang-Mills theory; Feynman rules are given and the standard uv -behaviour is recovered. In this formulation new non local observables can be introduced following the topological theory and giving an explicit realization of 't Hooft algebra.

1 The BF formulation of Yang-Mills theory

We consider the following euclidean first order action functional^{1,2,3}

$$S_{BFYM} = \int_{M^4} d^4x \frac{i}{2} \varepsilon^{\mu\nu\alpha\beta} B_{\mu\nu}^a F_{\alpha\beta}^a + g^2 \int_{M^4} d^4x B_{\mu\nu}^a B^{a\mu\nu} \quad , \quad (1)$$

where $F_{\mu\nu}^a \equiv 2\partial_{[\mu} A_{\nu]}^a + f^{abc} A_\mu^b A_\nu^c$ is the field strenght of the gauge field and where B is a Lie valued 2-form; the field equations of (1) are

$$*F_{\mu\nu}^a = 2ig^2 B_{\mu\nu}^a \quad , \quad \varepsilon^{\mu\nu\alpha\beta} D_\nu B_{\alpha\beta}^a = 0 \quad , \quad (2)$$

where $*F_{\mu\nu} \equiv \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta}$ is the "dual" field strength. This action enjoys the usual gauge invariance $\delta A_\mu = D_\mu c$, $\delta B_{\mu\nu} = i[c, B_{\mu\nu}]$ with $D_\mu \equiv \partial_\mu - i[A_\mu, \cdot]$ and the standard Yang-Mills (YM) action is recovered performing path integration over B or by substituting the field equations in it.

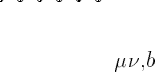
The first term in the r.h.s. of (1) is the action of a topological quantum field theory, the so-called BF theory^{4,5}. The pure BF theory has an extra "topological" symmetry

$$\delta A_\mu^a = 0 \quad , \quad \delta B_{\mu\nu}^a = 2\partial_{[\mu} \psi_{\nu]}^a + 2f^{abc} A_{[\mu}^b \psi_{\nu]}^c \quad , \quad (3)$$

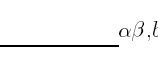
where ψ is a 1-form. Note that due to zero modes $\delta\psi_\mu = D_\mu \phi$ with $\delta\phi = 0$ in the transformations (3) the symmetry is reducible, allowing for a ghosts structure. The presence of this topological symmetry cancels out any local dynamics from the pure BF theory. In our case local degrees of freedom are restored by the second term in r.h.s. of (1) which is an explicit symmetry breaking for the topological symmetry (3) as long as $g \neq 0$. Therefore in this formulation YM theory appears as a deformation of the topological field theory and we call it BFYM.

2 Perturbative formulation

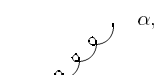
When casting a perturbative framework in BFYM we actually have two different ways of quantizing the action (1). The first one is to include the topological symmetry breaking term $g^2 B^2$ in the kinetic operator⁶, the second one is to regard it as a vertex. The latter procedure is quite involved because the kinetic operator in this case requires gauge fixing also for the topological symmetry and a ghosts of ghost structure; moreover the degrees of freedom of the topological group become dynamical and add to the field content of the theory. We will address to this case elsewhere⁷ and consider now the former one. The gauge fixing is straightforward and, after the field rescaling $B \rightarrow B/g$ and $A \rightarrow gA$, the following Feynman rules are easily obtained:




$$\Delta_{AA}(p)_{\mu\nu} = \delta^{ab} \frac{1}{p^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) ,$$



$$\Delta_{BA}^{ab}(p)_{\mu\nu\alpha} = -\frac{1}{2} \varepsilon_{\mu\nu\rho\alpha} \frac{p_\rho}{p^2} \delta^{ab} ,$$



$$\Delta_{BB}^{ab}(p)_{\mu\nu\alpha\beta} = -\frac{1}{4} \delta^{ab} \varepsilon^{\mu\nu\lambda\gamma} \varepsilon^{\alpha\beta\rho\gamma} \frac{p_\lambda p_\rho}{p^2} ,$$



$$\Lambda_{(BAA)}^{abc}{}_{\mu\nu\alpha\beta} = -i g f^{abc} \varepsilon_{\mu\nu\alpha\beta} .$$

Note the off-diagonal structure of the propagators. Indeed is this structure which recovers all the nonlinear interactions of YM, even if our theory has only

the trilinear vertex BAA . The ghost sector is unchanged in this formulation and its Feynman rules have to be added to the previous ones.

When studying one loop diagrams one finds in particular that the singular structure of the propagator Δ_{AA} is given in the Landau gauge by

$$\Delta_{AA}^{(1-loop)} = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4}$$

and requires for A the same wave function renormalization found in YM, i.e. $A_0 = Z_{AA}A_R$ with Z_{AA} given in dimensional regularization by $Z_{AA} = 1 + \frac{13}{12} \frac{c_V g^2}{16\pi^2 \epsilon}$. This value leads to the expected one loop β -function, $\beta_1 = \frac{11}{3} \frac{c_V}{16\pi^2}$, and therefore the uv -behaviour of YM and BFYM are the same⁶.

When performing the renormalization of the theory the dimensions and tensorial structure of the fields allow the operator mixing

$$\begin{pmatrix} B_0 \\ F_0 \end{pmatrix} = \begin{pmatrix} Z_{BB} & i * Z_{BA} \\ 0 & Z_{AA} \end{pmatrix} \cdot \begin{pmatrix} B_R \\ F_R \end{pmatrix} . \quad (4)$$

In this way is generated a counterterm $\sim F^2$, required to renormalize the non linear gluon vertices which arise at the one loop level. Note that $Z_{AB} \sim g^2 \frac{1}{\epsilon}$ in order to not modify Feynman rules at the tree level. After renormalization is performed is always possible to redefine B_R in order to reabsorb the F^2 term and recover the tree level structure of the theory.

3 A new observable

A new non-local observable associated to an orientable surface $\Sigma \in M^4$ is naturally introduced in the BF formulation of QCD^{1,2},

$$M(\Sigma, C) \equiv \text{Tr} \exp \left\{ i k \int_{\Sigma} d^2 y \text{Hol}_{\bar{x}}^y(\gamma) B(y) \text{Hol}_y^{\bar{x}}(\gamma') \right\} , \quad (5)$$

where $\text{Hol}_{\bar{x}}^y(\gamma)$ denotes the holonomy along the open path $\gamma \equiv \gamma_{\bar{x}y}$ with initial and final point \bar{x} and y respectively,

$$\text{Hol}_{\bar{x}}^y(\gamma) \equiv P \exp \left\{ i \int_{\bar{x}}^y dx^{\mu} A_{\mu}(x) \right\} . \quad (6)$$

In (5) k is an arbitrary parameter, \bar{x} is a *fixed* point and the relation between the assigned paths γ, γ' over Σ and the closed contour C is the following: C starts from the fixed point \bar{x} , connects a point $y \in \Sigma$ by the open path $\gamma_{\bar{x}y}$ and then returns back to the neighborhood of \bar{x} by $\gamma'_{y\bar{x}}$, (in general $(\gamma_{\bar{x}y})^{-1} \neq \gamma'_{y\bar{x}}$). From the neighborhood of \bar{x} the path starts again to connect another point $y' \in \Sigma$. Then it returns back and so on until all the points on Σ are connected. The path C is generic and no special ordering prescription is required. Of course the quantity (5) is path dependent and our strategy is to regard it as a loop variable once the surface Σ is given.

In particular it is possible to show¹ that M gives an explicit analytic realization of the 't Hooft loop operator⁸, the dual to the Wilson loop W . The *vev*'s of M and W label the phases of the theory and an explicit calculation in the BF formalism has shown the expected confining phase. Lastly, a quite similar formalism has been introduced recently in⁹ for the *vev* of W in a string picture of gauge theories.

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